中國文化大學 99 學年度轉學招生考試

系組：經濟學系二、資訊管理學系二年級　　日期節次：7 月 28 日第 3 節 13:30-14:50

科目：**微積分**　(141-108)

U-5-2

Transfer Test, Calculus

**Show all works to get full credits!**

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1.(15%) Evaluate the following integral

(5%) (a) $\int x \ln x \, dx$ (5%) (b) $\int x^2 e^{x^3+1} \, dx$ (5%) (c) $\int_2^\infty \frac{1}{x \ln x} \, dx$

2. (15%) Determine where the function

$$f(x) = 3x^4 - 2x^3 - 12x^2 + 18x + 15$$

is increasing and decreasing, and where its graph is concave up and concave down.

3. (15%) Find

$$\int\int_D 12x \, dA$$

where $R$ is the region bounded by $y = x^2$ and $y = 6 - x$.

4. (15%) Differentiate these functions

(7%) (a) $f(x) = e^{x^2} \ln(x^3 + 4)$ (8%) (b) $g(x) = \frac{\sqrt{x+3}}{(1-2x)^6}$

5. (15%) Determine whether each of the following series converges or diverges

(5%) (a) $\sum_{k=1}^{\infty} \frac{\ln k}{k}$ (5%) (b) $\sum_{k=3}^{\infty} \frac{k^2}{5^k}$ (5%) (c) $\sum_{k=1}^{\infty} (\frac{\pi}{2})^k$

6. (15%) Find all critical points for the function $f(x, y) = x^3 - y^3 + 6xy$ and classify each as a relative maximum, a relative minimum, or a saddle point.

7. (10%) Find the maximum and minimum values of the function

$$f(x, y) = xy$$

subject to the constraint $x^2 + y^2 = 8$.