

4-5-1

■不可使用計算機

一、Find each limit of the following: (10%)

$$(a) \lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x) \quad (b) \lim_{x \rightarrow 3} \frac{2x^2 - 7x - 15}{x - 5}$$

二、Function f is defined as follows: (10%)

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ ax + b & \text{if } x > 1 \end{cases}$$

Find value of a and b , such that f at $x = 1$ be differentiable.

$$\text{三、Find } \frac{dy}{dx}, \text{ if } y = \frac{(x^2 + 3)^{\frac{2}{3}} (3x + 2)^2}{\sqrt{x + 1}} \quad (10\%)$$

四、An open box is to be made from a square sheet of paper 12 centimeters on a side by cutting small squares from each of the corners and turning up the edges. What are the dimensions of the resulting box if its volume is to be a maximum? (10%)

$$\text{五、Evaluate } \lim_{n \rightarrow \infty} \left(1 + \frac{r}{n}\right)^{nt}, \text{ where } r \text{ and } t \text{ are constant (10\%)}$$

六、Find the area of the region between the curves $y = x^4$ and $y = 2x - x^2$. (10%)

七、Find each integral of the following: (10%)

$$(a) \int 2x - x^2 - x^4 dx \quad (b) \int x \ln x dx$$

$$\text{八、Evaluate the improper integral } \int_0^3 \frac{1}{(x-1)^2} dx \quad (10\%)$$

九、Use the method of Lagrange multipliers to find the minimum of the function (20%)

$$f(x, y, z) = 2xy + 6yz + 8xz$$

Subject to the constraint function $xyz = 12000$.