

中國文化大學 99 學年度轉學招生考試

系組：應用數學系二、物理學系二、化學系二、大氣科學系二年級

日期節次：7 月 27 日第 3

節 13:30-14:50

科目：微積分 (119-20)

U-3-1

一、填充題，每格 5 分，共 60 分。(請將答案依照題號寫在答案紙上，不需寫出計算過程。)

1. $\lim_{x \rightarrow 7} \frac{\sqrt{x+2}-3}{x-7} = \underline{\hspace{2cm}}$.

2. $\lim_{x \rightarrow -\infty} \frac{2x+1}{\sqrt{x^2+3}} = \underline{\hspace{2cm}}$.

3. $\frac{d}{dx} x^x = \underline{\hspace{2cm}}$.

4. $\int_0^3 \frac{dx}{x-1} = \underline{\hspace{2cm}}$.

5. $\int_0^{1/\sqrt{3}} \frac{dx}{1+3x^2} = \underline{\hspace{2cm}}$.

6. $\int_0^2 \int_{y/2}^1 e^{x^2} dx dy = \underline{\hspace{2cm}}$.

7. If $F(x) = \int_1^x f(t) dt$, where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u} du$, find $F''(2) = \underline{\hspace{2cm}}$.

8. The area of the region enclosed by $x = y^2$ and $y = x - 2$ is $\underline{\hspace{2cm}}$.

9. If $f(x) = x^5 + 2x + 1$, then $(f^{-1})'(4) = \underline{\hspace{2cm}}$.

10. The slope of the cycloid $x = 2(\theta - \sin \theta)$, $y = 2(1 - \cos \theta)$ at the point where $\theta = \frac{\pi}{3}$ is $\underline{\hspace{2cm}}$.

11. The interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)2^n}$ is $\underline{\hspace{2cm}}$.

12. The directional derivative of $f(x, y) = x^2 y^3 - 4y$ at $P_0(2, -1)$ in the direction $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j}$ is $\underline{\hspace{2cm}}$.

二、計算證明題，每題 10 分，共 40 分。(請寫下計算證明過程，否則不予計分。)

1. Show that the equation $x^3 + 2x - 1 = 0$ has exactly one real solution.

2. Find the Taylor series generated by $f(x) = e^{x^2}$ at $a = 0$. Then, approximate the integral $\int_0^1 e^{x^2} dx$ by using the Taylor polynomial of order 2.

3. Find all relative extrema and saddle points of $f(x, y) = 3x^3 + y^2 - 9x + 4y$.

4. Evaluate the integral $\iint_R e^{\frac{x+y}{x-y}} dA$, where R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, -2)$, and $(0, -1)$.