

## 中國文化大學 100 學年度轉學招生考試

系組：物理學系三年級

日期節次：7月26日第2節 11:00-12:20

科目：力學 (11-75)

1. A particle of mass  $m$  moves in the  $x-y$  plane so that its position vector is  $\vec{r} = r\cos(\omega t)\hat{i} + r\sin(\omega t)\hat{j}$ , where  $r$  and  $\omega$  are constants. Find the velocity and the acceleration. (15%)
2. The position of a particle is given by  $\vec{r} = A(e^{\alpha t}\hat{i} + e^{-\alpha t}\hat{j})$ , where  $\alpha$  is a constant. Find the velocity and the acceleration. (15%)
3. A particle of mass  $m$  moves in the  $x-y$  plane so that its position vector is  $\vec{r} = a\cos(\omega t)\hat{i} + b\sin(\omega t)\hat{j}$ , where  $a$ ,  $b$  and  $\omega$  are constants and  $a > b$ . (a) Show that the particle moves in an ellipse. (b) Show that the force acting on the particle is conservative. (20%)
4. Show by means of the substitution  $r = \frac{1}{u}$  that the differential equation  $m(\ddot{r} - r\dot{\theta}^2) = f(r)$  for the path of the particle in a central field can be written as

$$\frac{d^2u}{d\theta^2} + u = -\frac{1}{ml^2u^2} f\left(\frac{1}{u}\right). \quad (20\%)$$

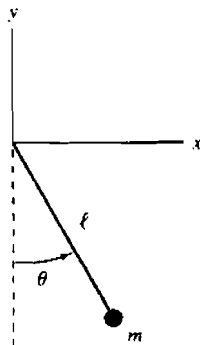


Fig.-1

5. As shown in Figure-1, use the  $(x,y)$  coordinate system to find (a) the kinetic energy  $T$ , the potential energy  $U$ , and the Lagrangian  $L$  for the case of a simple pendulum (with length  $l$ , mass  $m$ ) which is moving in a uniform gravitational field. Using Lagrangian formalism to find the equation of motion (b) in the  $(x,y)$  coordinate and (c) in the polar coordinate  $(r,\theta)$ . (30%)