中國文化大學 99 學年度轉學招生考試

系組：應用數學系三年級　　日期節次：7 月 27 日第 4 節 15:20-16:40

科目：**高等微積分**　(141-75)

Transfer Test; Advanced Calculus; Show all works to get full credits!

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1. (15%) **State** the following theorems

5% (a) Bolzano-Weierstrass theorem

5% (b) Heine-Borel theorem

5% (c) Weierstrass M-test

2. (15%) **Prove or disprove** that $f(x) = x^2$ is uniformly continuous on the real line $\mathcal{R}$.

3. (15%)

5% (a) Let $a < b$ and $f : [a, b] \to R$ be bounded. Given the definition that "f is Riemann integrable on [a,b]".

10% (b) If f is Rieman integrable on [a,b], then **prove**

$$F(x) = \int_a^x f(t)\,dt$$

exists and is continuous on [a,b].

4. (15%) **Show** the following function converges or has no limit as $(x, y) \to (0, 0)$.

$$(a)\quad f(x,y) = \frac{3x^2y}{x^2+y^2} \qquad (b)\quad g(x,y) = \frac{2xy}{x^2+y^2}$$

5. (15%) Let $\mathcal{E}$ be a nonempty subset of the real line R and suppose that $f_n \to f$ uniformly on $\mathcal{E}$. If each f_n is continuous at some $x_0 \in \mathcal{E}$, then **prove** that f is continuous at $x_0 \in \mathcal{E}$.

6. (10%) Let $H \subseteq R^n$ be a nonempty connected set. If f is continuous on H, then **prove** $f(H)$ is also connected.

7. (15%)

(a) Assume $f : R^n \to R^m$ is a function and $a \in R^n$. Give the definition of "f is differentiable at a".

(b) Is this function

$$f(x,y) = \begin{cases} \frac{y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

differentiable at (0,0)?