

中國文化大學 99 學年度轉學招生考試

系組：應用數學系三年級

日期節次：7 月 27 日第 3 節 13:30-14:50

科目：線性代數 (19-21)

試題中： $A, B, C$  皆代表矩陣。 $A^T$  代表  $A$  的轉置矩陣。 $A^{-1}$  代表  $A$  的反矩陣。

1~6 題為簡答題，每題 5 分，只須回答 True 或 False：

1. If  $AB = I$  and  $BC = I$ , then  $A = C$ .
2. If  $A$  is 3 by 4 and  $B$  is 4 by 5, then  $(AB)^T$  is 5 by 3.
3. Let  $A$  and  $B$  be 2 matrices, then  $(A - B)^2 = A^2 - 2AB + B^2$ .
4. If  $A$  and  $B$  are symmetric, then the transpose of  $AB$  is  $BA$ .
5.  $A$  and  $-A$  always have the same reduced echelon form.
6. If  $A$  and  $B$  are permutation matrices, then  $AB = BA$ .

7~13 題為計算證明題，每題 10 分，必須寫清楚每一題的詳細過程：

7. Let  $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ . Find  $A^3$ .

8. Let  $A$  and  $B$  be symmetric. Prove  $ABA$  is also symmetric.

9. Let  $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 6 & 8 \end{bmatrix}$ . Find the rank of  $A$ .

10. Let  $A = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 0 & 1 & 1 & 1 \\ 2 & 6 & 4 & 8 \end{bmatrix}$ . Find the dimension of the row space of  $A$ .

11. Let  $A = \begin{bmatrix} 9 & 12 \\ 12 & 16 \end{bmatrix}$ . If  $B^{-1}AB$  is a diagonal matrix, find  $B$ .

12. Prove: If  $A$  and  $B$  are positive definite, then  $A + B$  is also positive definite.

13. Prove or disprove:  $\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$  is similar to  $\begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix}$ .