

題目中, A, B 皆代表矩陣. A^T 代表 A 的轉置矩陣. A^{-1} 代表 A 的反矩陣.

1~6 題為簡答題, 每題 5 分, 只須回答 True 或 False :

1. Let A and B be 2 matrices, then $(A+B)^2 = A^2 + 2AB + B^2$.
2. If A is invertible and $AB = AC$, then $B = C$.
3. If A is not symmetric, then A^{-1} is not symmetric.
4. The columns of a matrix are a basis for the column space.
5. AB and BA have the same determinant.
6. Every positive definite matrix is invertible.

7~13 題為計算證明題, 每題 10 分, 必須寫清楚每一題的詳細過程 :

7. Let A be a 2 by 2 matrix such that $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$. Find A .

8. There are sixteen 2 by 2 matrices whose entries are 1's and 0's. How many of them are invertible?

9. A permutation matrix has the rows of identity matrix I in any order. Find a 3 by 3 permutation matrix P with $P^3 = I$ but $P \neq I$.

10. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 & 4 \\ 3 & 9 & 6 \end{bmatrix}$. Find the rank of AB .

11. A 3 by 3 matrix B is known to have eigenvalues 0, 1, 2. Find the determinant of $B^T B$.

12. Prove that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\begin{bmatrix} d & c \\ b & a \end{bmatrix}$ are similar.

13. Suppose a linear T transforms $(1, 1)$ to $(2, 2)$ and $(2, 0)$ to $(0, 0)$. Find $T(v)$ when $v = (-1, 1)$.