

題目中, A, B 皆代表矩陣. A^T 代表 A 的轉置矩陣. A^{-1} 代表 A 的反矩陣.

1~6 題為簡答題, 每題 5 分, 只須回答 True 或 False :

1. Let A and B be 2 matrices, if $AB = B$, then $A^2 = A$.
2. If A and B are 2 matrices, then $(A - B)^2 = (B - A)^2$.
3. If A is an invertible matrix, then A^2 is invertible.
4. If A and B are symmetric matrices, then AB is symmetric.
5. If A and B are 2 matrices, then rank of $AB \leq$ rank of A .
6. If $A^T = -A$, then the row space of $A =$ the column space of A .

7~13 題為計算證明題, 每題 10 分, 必須寫清楚每一題的詳細過程:

7. Let $A = \begin{bmatrix} 0 & 3 \\ 4 & 0 \end{bmatrix}$. Find A^{-1} .

8. $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$. Find A^{-1} .

9. Let $A = \begin{bmatrix} c & 5 \\ 5 & c \end{bmatrix}$ and rank of $A \neq 2$. Find c .

10. Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$. Find a basis for the column space of A .

11. $\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} = S \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} S^{-1}$. Find the matrix S .

12. Prove: if A is similar to B , then A^2 is similar to B^2 .

13. Prove: A and A^T have the same number of pivots.