

中國文化大學 97 學年度轉學招生考試

系組：應用數學系二年級 日期節次：7 月 29 日第 3 節

科目：微積分 (123-22)

一、填充題，每格 5 分，共 60 分。(請將答案依照題號寫在答案紙上，不需寫出計算過程。)

1. $\lim_{x \rightarrow 2} \frac{\cos(\pi/x)}{x-2} = \underline{\hspace{2cm}}$.

2. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{\pi}{4n} \sec^2\left(\frac{\pi k}{4n}\right) = \underline{\hspace{2cm}}$.

3. $\frac{d}{dx} \int_{\sqrt{x}}^0 t \sec t dt = \underline{\hspace{2cm}}$.

4. $\int_0^{\pi/2} \sin^3 x \cos^2 x dx = \underline{\hspace{2cm}}$.

5. $\int_0^{1/\sqrt{3}} \frac{dx}{1+3x^2} = \underline{\hspace{2cm}}$.

6. $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy = \underline{\hspace{2cm}}$.

7. The equation of the line tangent to the curve $x^3 + y^3 = 3xy$ at $P_0(x, y) = P_0(3/2, 3/2)$ is $\underline{\hspace{2cm}}$.8. The area of the region bounded by the curves $y = \sqrt{x}$ and $y = x$ is $\underline{\hspace{2cm}}$.9. The total arc length of the cardioid $r = 1 + \cos \theta$ is $\underline{\hspace{2cm}}$.10. If $f(x) = x^3 + 2x - 1$, then $(f^{-1})'(2) = \underline{\hspace{2cm}}$.11. The directional derivative of $f(x, y) = e^{xy}$ at $P_0(-2, 0)$ in the direction $\mathbf{v} = \mathbf{i} + \sqrt{3}\mathbf{j}$ is $\underline{\hspace{2cm}}$.12. The interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^n}{3^n}$ is $\underline{\hspace{2cm}}$.

二、計算證明題，每題 10 分，共 40 分。(請寫下計算證明過程，否則不予計分。)

1. Give the precise definition of $\lim_{x \rightarrow a} f(x) = L$. Then use $\varepsilon - \delta$ method to prove that $\lim_{x \rightarrow 3} x^2 = 9$.2. State the **Mean Value Theorem** and show that if $0 < x < y$, then $\sqrt{y} - \sqrt{x} < \frac{y-x}{2\sqrt{x}}$.3. Find the Taylor series generated by $f(x) = \sin(x^2)$ at $x = 0$. Then, use the Taylor polynomial of order 3 to approximate the integral $\int_0^1 \sin(x^2) dx$.4. Find all relative extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$.