1.(20%) Determine whether the following sets are open, closed, compact, or connected.

(1) $Q = \{$ the rational number in $R\}$.

(2) $E = \{\frac{1}{n} \in R \mid n \in N\} \cup \{0\}$.

2.(10%) Write down the negation of the following statement.

" $\forall \varepsilon > 0, \exists \delta > 0, \ni \forall x, y \in [0,1]$, if $|x-y| < \delta$, then $|f(x) - f(y)| < \varepsilon$ "



3. (20%) Prove that for fixed $c$, $c \in (0,1)$, the sequence $\{nc^n\}$ converges to 0 by $\varepsilon - N$ definition.

4.(15%) Prove that $\sum_{n=1}^{\infty} \frac{(\sin nx)^2}{n^2}$ converges uniformly on $R$.

5.(20%) Does $\frac{d}{dx} \sum_{n=1}^{\infty} \frac{1}{x+n^2} = \sum_{n=1}^{\infty} \frac{d}{dx} \frac{1}{x+n^2}$ on $[0, \infty)$. Prove your answer.

6.(15%) Let $f: R^2 \to R$ be defined by $f(x,y) = \frac{xy}{x^2+y^2}$, for $(x,y) \neq (0,0)$, and $f(0,0) = 0$. Is $f$ differentiable at (0,0)? Prove your answer.