

1~6 題爲簡答題，每題 5 分，每題只須回答 True 或 False：

1. If A is an $n \times n$ matrix and $A^2 = 0$, then $A = 0$.
2. Let A and B be 2 matrices, then $(AB)^T = A^T B^T$.
3. If A and B are symmetric, then $(A+B)(A-B)$ is symmetric.
4. Let A and B be 2 $n \times n$ invertible matrices, then AB and B have the same nullspace.
5. Let A and B be 2 $n \times n$ matrices. If A is not invertible, then AB is not invertible.
6. If A is similar to B , then A and B have the same eigenvectors.

7~13 題爲計算證明題，每題 10 分，必須寫清楚每一題的詳細過程：

7. Let A be a 2×2 matrix such that $A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ -x \end{bmatrix}$. Find A .

8. Let A be a 2×2 matrix such that $A \neq I$ and $A^{-1} = A$.

Find A . (I 表示單位矩陣)

9. Find the rank of $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 2 & 8 & 10 \\ 3 & 3 & 10 & 13 \end{bmatrix}$.

10. Find a basis for the nullspace of $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$.

11. $A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$. Find $\lim_{k \rightarrow \infty} A^k$.

12. Prove: If A^{-1} is positive definite, then A is positive definite.

13. Prove: Orthonormal vectors are linearly independent.