

試題中， A, B 代表矩陣。 A^T 代表 A 的轉置矩陣。 A^{-1} 代表 A 的反矩陣。

1~4 題為簡答題，每題 5 分，只須回答 True 或 False：

1. If A is n by n , then A and A^{-1} have the same determinant.

2. If A is invertible, then $\det(A^2) = (\det A)^2$.

3. If A^{-1} is positive definite, then A is also positive definite.

4. If A and B are 3 by 3 matrices, then $(AB)^T = A^T B^T$.

5~12 題為計算證明題，每題 10 分，必須寫清楚每一題的詳細過程：

5. $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find A^{-1} .

6. Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$. Find a basis for the row space of A .

7. Are the vectors $(1,2,1)$, $(0,1,2)$ and $(3,8,7)$ linearly independent?

8. $A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 2 \end{bmatrix}$ Find the dimension of the column space of A .

9. Is $\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ diagonalizable (可對角化)?

10. Is $\begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}$ positive definite?

11. Is $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ similar to $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$?

12. Prove: If A is a 2 by 2 projection matrix, then the eigenvalues of A are 0 and 1.