

試題中， $A, B$ 代表矩陣。 $A^T$ 代表 $A$ 的轉置矩陣。 $A^{-1}$ 代表 $A$ 的反矩陣。

1~4 題為簡答題，每題 5 分，只須回答 True 或 False：

1. If  $A$  is  $n$  by  $n$ , then  $A$  and  $A^T$  have the same determinant.  
行列式
2. If  $A, B$ , and  $C$  are matrices, then  $(AB)C = A(BC)$ .
3. If  $A$  and  $B$  are invertible, then  $A+B$  is invertible.
4. If  $A$  and  $B$  are symmetric, then  $ABAB$  is symmetric

5~12 題為計算證明題，每題 10 分，必須寫清楚每一題的詳細過程：

5. Find a 2 by 2 matrix  $A$  such that  $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ .

6. Let  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ . Find  $A^{-1}$ .

7. Let  $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$ . Find the rank of  $A^T$ .

8. Find a basis for the nullspace of  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ .

9. Prove or disprove: The columns of a matrix are a basis for the column space.

10. Prove that if  $A$  and  $B$  are orthogonal matrices, then  $AB$  is also an orthogonal matrix.

11. Let  $A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$ . Find the eigenvalues and eigenvectors of  $A$ .

12. Prove: If  $A$  is positive definite, then  $A^{-1}$  is also positive definite.