中國文化大學 100 學年度轉學招生考試

系組：應用數學系三年級　　日期節次：7 月 26 日第 2 節 11:00-12:20

科目：高等微積分　(112-74)

1.(15%) True or False:

a) $\mathbf{Q}$ is dense in $\mathbf{R}$.
b) $\mathbf{R}$ is complete.
c) Every single point in an Euclidean n-space is a closed set.
d) A connected set is also path-connected.
e) Every integrable function is continuous.

2.(15%) Determine which of the following are open, closed, or compact in $\mathbf{R}^2$:

$A = \{(x,y) |\ 1 < x < 2\ \&\ 2 < y < 3\}$
$B = \{(x,y) |\ x^2 + y^2 \leq 1\} \cup \{(0,2),(2,1)\}$
$C = \{(x,y) |\ x \in \mathbf{Z}\ \&\ y \in \mathbf{Z}\}$

3.(15%) Show that a set A in a metric space M is closed if and only if the accumulation points of A belongs to A.

4.(10%) Show that if d is a metric on a set M, and ρ is defined by

$$\rho(x,y) = \frac{d(x,y)}{1 + d(x,y)},$$

then ρ is also a metric on M.

5.(15%) Let $f_n : \mathbf{R} \to \mathbf{R}$ be uniformly continuous and let f_n converge uniformly to f. Must f be uniformly continuous?

6.(15%) Let

$$f_n(x) = \frac{x}{1 + nx^2}.$$

Show that f_n converges uniformly on $\mathbf{R}$.

7.(15%) Let

$$f(x,y) = \frac{x^2 y^2}{\sqrt{x^2 + y^2}}, (x,y) \neq (0,0)$$

and

$$f(x,y) = 0, (x,y) = (0,0).$$

Is f differentiable at $(0,0)$?