

中國文化大學 100 學年度轉學招生考試

系組：應用數學系二年級

日期節次：7 月 26 日 第 1 節 09:00-10:20

科目：微積分 (122-22)

0-3-1

一、單選題，每題 3 分，共 30 分。(請將答案依照題號寫在答案紙上，不需寫出計算過程。)

1. ( ) Us the fact that  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$  to find the value of  $\lim_{x \rightarrow 0} \frac{2x}{\tan 7x}$ .
- (A)  $\infty$  (B) 0 (C) 1 (D)  $\frac{2}{7}$  (E)  $\frac{7}{2}$ .
2. ( )  $\int_{-\infty}^{\infty} x e^{-x^2} dx =$  (A) 0 (B) 1 (C)  $\frac{1}{2}$  (D)  $\frac{-1}{2}$  (E) None of these.
3. ( ) The slope of the normal (法線斜率) to the curve  $x^3 + y^3 - 6xy = 1$  at the point  $P(0, 1)$  is
- (A) 1 (B) -1 (C) 2 (D)  $\frac{-1}{2}$  (E) None of these.
4. ( ) The graph of the polar equation  $r^2 = 4r \cos \theta$  is a
- (A) cardioids(心臟線) (B) line(直線) (C) spiral(螺線) (D) roses(玫瑰線) (E) circle(圓).
5. ( ) The volume of the solid generated by revolving the region bounded by  $y = \sqrt{x}$ ,  $0 \leq x \leq 4$  and the x-axis is (A)  $\frac{16\pi}{3}$  (B)  $2\pi$  (C)  $8\pi$  (D)  $\frac{\pi}{4}$  (E) None of these.
6. ( ) If  $f$  is continuous on  $[0, 1]$ , then  $\int_0^1 f(x) dx =$
- (A)  $\int_0^1 f(1-x) dx$  (B)  $\int_0^1 f(1+x) dx$  (C)  $\int_0^1 f(x-1) dx$  (D)  $\int_1^0 f(x-1) dx$  (E) None of these.
7. ( ) Which of the following series converges?
- (A)  $\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 1}$  (B)  $\sum_{n=1}^{\infty} 2^n$  (C)  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$  (D)  $\sum_{n=2}^{\infty} \frac{1}{\ln n}$  (E) None of the above.
8. ( )  $\int_0^2 \sqrt{4-x^2} dx = ?$  (A)  $\frac{\pi}{2}$  (B)  $\pi$  (C)  $2\pi$  (D)  $\frac{2\pi}{3}$  (E) 4.
9. ( ) If  $f(x) = x^5 + 2x + 1$ , then  $(f^{-1})'(4) =$  (A)  $\frac{1}{5}$  (B)  $\frac{1}{6}$  (C)  $\frac{1}{7}$  (D)  $\frac{1}{8}$  (E)  $\frac{1}{9}$ .
10. ( ) The average value of  $f(x) = 1 + x^2$  on the interval  $[-1, 2]$  is
- (A) 2 (B) 3 (C) 4 (D) 6 (E) None of these

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二、填充題，每格 5 分，共 40 分。(請將答案依照題號寫在答案紙上，不需寫出計算過程。)

1.  $\lim_{x \rightarrow 0} \frac{|x|+3x}{2x} = \underline{\hspace{2cm}}$ .

2.  $\frac{d}{dx} \int_{\sqrt{x}}^1 \sin(t^2) dt = \underline{\hspace{2cm}}$ .

3.  $\int_0^3 x\sqrt{1+x} dx = \underline{\hspace{2cm}}$ .

4.  $\int_0^1 \int_x^1 \sin(y^2) dy dx = \underline{\hspace{2cm}}$ .

5. The area of the region enclosed by  $x = y^2$  and  $y = x - 2$  is  $\underline{\hspace{2cm}}$ .6. The slope of the cycloid  $x = 2(t - \sin t)$ ,  $y = 2(1 - \cos t)$  at the point where  $t = \frac{\pi}{3}$  is  $\underline{\hspace{2cm}}$ .7. The interval of convergence of the power series  $\sum_{n=0}^{\infty} \frac{4^n x^{2n}}{n}$  is  $\underline{\hspace{2cm}}$ .8. The directional derivative of  $f(x, y, z) = x^3 - xy^2 - z$  at  $P_0(1, 1, 0)$  in the direction  $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$  is  $\underline{\hspace{2cm}}$ .

三、計算證明題，每題 10 分，共 30 分。(請寫下計算證明過程，否則不予計分。)

1. Use  $\varepsilon - \delta$  method to prove  $\lim_{x \rightarrow 3} x^2 = 9$ .2. Find all relative extrema and saddle points of  $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$ .3. Evaluate the integral  $\int_0^1 \int_0^{1-x} \sqrt{x+y}(y-2x)^2 dy dx$ . (Hint: Let  $u = x + y$ ,  $v = y - 2x$ )

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