

系所(組)別：地學研究所大氣組 M-3-3 考試科目：應用數學

1(a) 何謂函數正交？(10%)

(b) Legendre Polynomials, $P_2(x) = (3x^2 - 1)/2$, $P_3(x) = (5x^3 - 3x)/2$ 證明 $P_2(x)$ 與 $P_3(x)$ 為正交 (10%)2(a) 函數 $f(x) = \begin{cases} 1, & \text{if } |x| < 1 \\ 0, & \text{if } |x| > 1 \end{cases}$, 求此函數之 Fourier Integral (10%)(b) 利用 2(a) 之結果求 $\int_0^{\infty} \frac{\sin w}{w} dw$ 之積分值 (10%)

3. Reduce to Bessel's equation and find the general solution

(a) $x^2 y'' + xy' + (\lambda^2 x^2 - \nu^2)y = 0$, hint ($\lambda x = z$) (10%)(b) $x^2 y'' + xy' + (x^2 - \frac{1}{16})y = 0$ (10%)

4. 求微分方程式之通解

(a) $y'' + y' + y = \sin(x)$ (10%)(b) $y' = 2 \sec(2y)$ (10%)5. Green's theorem in the plane $\iint_R \nabla \cdot \bar{F} dx dy = \oint \bar{F} \cdot \hat{n} ds$ 證明 $\iint_R \nabla^2 w dx dy = \oint \frac{\partial w}{\partial n} ds$ (20%)